

68. If $y = \sin^{-1}(x^2 \sqrt{1-x^2} + x\sqrt{1-x^4})$, then prove that $\frac{dy}{dx} = \frac{2x}{\sqrt{1-x^4}} + \frac{1}{\sqrt{1-x^2}}$.
69. If $\sqrt{1-x^6} + \sqrt{1-y^6} = a^3(x^3 - y^3)$, prove that $\frac{dy}{dx} = \frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}$.
70. If $x^y = e^{y \log x}$, show that $\frac{dy}{dx} = \frac{2 - \log x}{(1 - \log x)^2}$.
71. If $\sin y = x \cos(a + y)$, show that $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\cos a}$.
Also, show that $\frac{dy}{dx} = \cos a$, when $x = 0$. [Delhi 2018(C)]
72. If the derivative of $\tan^{-1}(a + bx)$ takes the value 1 at $x = 0$, prove that $b = 1 + a^2$.
73. If $y = e^{x + e^{x + e^{x + \dots}}}$, prove that $\frac{dy}{dx} = \frac{y}{1-y}$.
74. Differentiate $\tan^{-1} \frac{2x}{1-x^2}$ with respect to $\sin^{-1} \frac{2x}{1+x^2}$.
75. If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin x^2$, find $\frac{dy}{dx}$. [HOTS]
76. If $y = \sin(m \sin^{-1} x)$, prove that $(1-x^2)y_2 - xy_1 + m^2y = 0$.
77. Differentiate $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, w.r.t. $\tan^{-1} \frac{x}{\sqrt{1-x^2}}$.
78. If $y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$, show that $\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} - x\right) = 0$.
79. If $x^y + y^x = 2$ (or a or b or $a + b$ in place of 2), find $\frac{dy}{dx}$.
80. Find $\frac{dy}{dx}$, if $\tan(x + y) + \tan(x - y) = 1$. [HOTS]
81. If $x^y y^x = 1$, find $\frac{dy}{dx}$.
82. If $x \sin(a + y) + \sin a \cos(a + y) = 0$, prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$. [NCERT Exemplar]
83. If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{x-y}{x \log x}$. [NCERT]
84. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$. [NCERT Exemplar, HOTS]
85. If $\cos y = x \cos(a + y)$ with $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}$. [NCERT; Foreign 2014]
86. Verify the Rolle's Theorem for the function $f(x) = \sin^4 x + \cos^4 x$ in $\left[0, \frac{\pi}{2}\right]$.
87. Verify the Rolle's Theorem for the function $f(x) = \sin 3x$ in $[0, \pi]$.
88. Verify the Rolle's Theorem for the function $f(x) = \sqrt{4-x^2}$ in $[-2, 2]$. [NCERT Exemplar]
89. Verify Mean Value Theorem for the function $f(x) = \sqrt{25-x^2}$ in $[-3, 4]$.
90. If $f(x)$ and $g(x)$ are functions derivable in $[a, b]$ such that $f(a) = 4, f(b) = 10, g(a) = 1, g(b) = 3$, show that for $a < c < b$, we have $f'(c) = 3g'(c)$. [HOTS]
91. Examine the following function $f(x)$ for continuity at $x = 1$ and differentiability at $x = 2$.
$$f(x) = \begin{cases} 5x-4 & , 0 < x < 1 \\ 4x^2-3x & , 1 < x < 2 \\ 3x-4 & , x \geq 2 \end{cases}$$
 [Gwahati 2015]
92. If $y = x^3 \log\left(\frac{1}{x}\right)$, then prove that $x \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 3x^2 = 0$. [Gwahati 2015]
93. If $\frac{x}{x-y} = \log \frac{a}{x-y}$, then prove that $\frac{dy}{dx} = 2 - \frac{x}{y}$. [Gwahati 2015]
94. Let $f(x) = x - |x - x^2|, x \in [-1, 1]$. Find the point of discontinuity, (if any), of this function in $[-1, 1]$. [Bhubaneswar 2015]
95. If $y = \log\left(\frac{x}{a-bx}\right)^2$, prove that $x^3 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y\right)^2$. [Bhubaneswar 2015]
96. Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1-x^2}$ at $x = \frac{1}{2}$. [Bhubaneswar 2015]
97. If $x = \alpha \sin 2t(1 + \cos 2t)$ and $y = \beta \cos 2t(1 - \cos 2t)$, show that $\frac{dy}{dx} = \frac{\beta}{\alpha} \tan t$. [Patna 2015]
98. Find $\frac{d}{dx} \cos^{-1}\left(\frac{x-x^{-1}}{x+x^{-1}}\right)$. [Patna 2015]
99. Find the derivative of the following function $f(x)$ w.r.t. x , at $x = 1$: $\cos^{-1}\left[\sin \sqrt{\frac{1+x}{2}}\right] + x^2$. [Patna 2015]
100. If function $f(x) = |x-3| + |x-4|$, then show that $f(x)$ is not differentiable at $x = 3$ and $x = 4$. [Chennai 2015]
101. If $y = x^{e^{-x^2}}$, find $\frac{dy}{dx}$. [Chennai 2015]
102. If $y = \sqrt{x+1} - \sqrt{x-1}$, prove that $(x^2-1) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - \frac{1}{4}y = 0$. [Chennai 2015]

32. $y = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$.
33. $y = \cos^{-1}\left(\frac{3x + 4\sqrt{1-x^2}}{5}\right)$.
34. $f(x) = \tan^{-1}\left(\frac{1-x}{1+x}\right) - \tan^{-1}\left(\frac{x+2}{1-2x}\right)$. [HOIS]
35. $y = \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$. [HOIS]
36. $y = \sin^{-1}\left[\frac{\sqrt{1+x} + \sqrt{1-x}}{2}\right]$.
37. $y = \sin^{-1}[2ax\sqrt{1-a^2x^2}]$.
38. $y = \cot^{-1}\left(\frac{1+x}{1-x}\right)$.
39. $y = \tan^{-1}\left[\frac{\sqrt{1+a^2x^2}-1}{ax}\right]$. [HOIS]
40. $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right], x^2 \leq 1$. [Delhi 2015]
41. Differentiate $\cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right], 0 < x < \frac{\pi}{2}$. [NCERT]
42. Differentiate $\sec^{-1}\left(\frac{1}{4x^3-3x}\right), 0 < x < \frac{1}{\sqrt{2}}$ w.r.t. x . [NCERT Exemplar]
43. Prove that $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right] = \sqrt{a^2-x^2}$. [Foreign 2011]
44. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}(2x\sqrt{1-x^2})$, when $x \neq 0$. [Delhi 2014]
45. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, when $x \neq 0$. [Delhi 2014]
46. Find $\frac{dy}{dx}$, if $y = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$.
47. If $x = \cos t(3-2\cos^2 t)$ and $y = \sin t(3-2\sin^2 t)$, find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$. [AI 2014]
48. If $x = a \cos \theta + b \sin \theta$ and $y = a \sin \theta - b \cos \theta$, show that $y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$. [Delhi 2015, Foreign 2014]
49. If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx} + e^{y-x} = 0$. [Foreign 2014]
50. If $f(x) = \sqrt{x^2+1}$; $g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x-3$, then find $f'[h'\{g'(x)\}]$. [Foreign 2015]
51. If $y = (x + \sqrt{1+x^2})$, then show that $(1+x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = n^2y$. [Foreign 2015]
52. Find $\frac{dy}{dx}$, if $y = \cos(\log x)^2$. [HOIS]
53. Find $\frac{dy}{dx}$, if $y = (\cos x)^x + (\sin x)^{\frac{1}{x}}$.
54. Find $\frac{dy}{dx}$, if $y = x^{\sin x - \cos x} + \frac{x^2-1}{x^2+1}$.
55. If $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a(1 + \sin t)$, find $\frac{d^2y}{dx^2}$.
56. If $x = a(\theta - \sin \theta)$, $y = a(1 + \cos \theta)$, find $\frac{d^2y}{dx^2}$. [Delhi 2011]
57. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, $0 < t < \frac{\pi}{2}$, find $\frac{d^2x}{dt^2}$, $\frac{d^2y}{dt^2}$, and $\frac{d^2y}{dx^2}$. [Delhi 2017(C); AI 2012]
58. If $x = a \sin t$ and $y = a(\cos t + \log \tan \frac{t}{2})$, find $\frac{d^2y}{dx^2}$. [Delhi 2013]
59. If $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$. [AI 2013]
60. If $x \log y + y \log x = 5$, find $\frac{dy}{dx}$.
61. If $\log(x^2+y^2) = 2 \tan^{-1}\left(\frac{y}{x}\right)$, then show that $\frac{dy}{dx} = \frac{x+y}{x-y}$. [Delhi 2019]
62. If $y = \log(x + \sqrt{x^2+1})$, then prove that $(x^2+1) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0$. [Delhi 2013; Foreign 2011]
63. If $y = \sin^{-1} x$, show that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0$. [NCERT; Delhi 2012]
64. If $y = 3 \cos(\log x) + 4 \sin(\log x)$, show that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$. [NCERT; AI 2016; Delhi 2012]
65. If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$. [AI 2013]
66. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$. [NCERT Exemplar; AI 2013]
67. Differentiate, $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1} x$, when $x \neq 0$. [NCERT Exemplar; Foreign 2013]

4. State which of the following is continuous as well as differentiable for $x \in R$

- (a) $|x|$ (b) $[x]$
 (c) polynomial function (d) $\text{sgn}(x)$

5. Derivative of $\frac{x}{x-1}$ with respect to x , is

- (a) 2 (b) $\frac{1}{(x-1)^2}$
 (c) $\frac{2x-1}{(x-1)^2}$ (d) $\frac{-1}{(x-1)^2}$

6. Derivative of $\sin x$ with respect to $\log x$, is

- (a) $\frac{x}{\cos x}$ (b) $\frac{\cos x}{x}$
 (c) $x \cos x$ (d) $x^2 \cos x$

7. If $y = \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} + \frac{1}{1+x^{b-a}+x^{c-a}}$, then find $\frac{dy}{dx}$.

8. Examine the continuity of the function $f(x) = \frac{1}{x+3}$, $x \in R$.

9. State the points of discontinuity for the function $f(x) = [x]$, in $-3 < x < 3$. [HOIS]

10. Find the point of discontinuity if any for the function $f(x) = \frac{1}{x-5}$. [NCERT]

11. Differentiate $y = e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$ with respect to x . [NCERT]

12. If $y = 500e^{7x} + 600e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$. [NCERT]

13. Verify the Rolle's Theorem for the function $f(x) = \sin x$ in $[0, \pi]$.

14. Verify the Rolle's Theorem for the function $f(x) = \frac{1}{x}$ in $[-1, 1]$. [HOIS]

15. Verify Mean Value Theorem for the function $f(x) = (x-1)^{2/3}$ in $[0, 2]$.

16. Find the derivative of $f(\log x)$ with respect to x , where $f(x) = \log x$.

17. Find the derivative of $\frac{\sin^{-1} x}{1 + \sin^{-1} x}$ with respect to $\sin^{-1} x$.

18. The derivative of a differentiable even function is odd function. State true or false.

19. It is known that for $x \neq 1$, $1 + x + x^2 + \dots + x^{n-1} = \frac{1-x^n}{1-x}$.

Hence find the sum of the series $1 + 2x + 3x^2 + \dots + (n-1)x^{n-2}$.

Long Answer I / Long Answer II Type

20. Find the relationship between a and b so that the function f defined by $f(x) = \begin{cases} ax+1, & \text{if } x \leq 3 \\ bx+3, & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$. [NCERT, XI 2011]

21. For what value of λ is the function defined by

$$f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases} \text{ continuous at } x = 0? \quad [\text{NCERT; Foreign 2011}]$$

22. Show that the function $f(x) = 2x - |x|$ is continuous but not differentiable at $x = 0$. [Foreign 2013]

23. Is the function $f(x) = \frac{3x + 4 \tan x}{x}$ continuous at $x = 0$? If not, how should we define the function to make it continuous?

24. If $f(x) = \begin{cases} [x], & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$, find whether $f(x)$ is continuous at $x = 0$. [HOIS]

25. Is the function $f(x) = \begin{cases} [x]-1, & \text{if } x \neq 1 \\ -1, & \text{if } x = 1 \end{cases}$ continuous at $x = 1$?

26. For what value of k is the following function continuous at $x = 0$?

$$f(x) = \begin{cases} \frac{\sin 5x}{3x}, & x \neq 0 \\ k, & x = 0. \end{cases} \quad [\text{HOIS}]$$

27. Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{\log(1+3x)}{x}, & \text{if } x \neq 0 \\ 3, & \text{if } x = 0 \end{cases} \text{ at } x = 0.$$

28. Show that the function defined by $f(x) = \cos x^2$ is a continuous function. [NCERT]

29. Find the value of k such that the function

$$f(x) = \begin{cases} 2^{x+2} - 16, & \text{if } x \neq 2 \\ k, & \text{if } x = 2 \end{cases} \text{ is continuous at } x = 2. \quad [\text{NCERT Exemplar}]$$

30. For what value of k , is the following function continuous at $x = 0$?

$$f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases} \quad [\text{NCERT Exemplar}]$$

Differentiate each of the following with respect to x (Exercises 31 to 40):

31. $y = \sin^{-1} \left(\frac{5x + 12\sqrt{1-x^2}}{13} \right)$